Risk Theory

Tutorial 1

ISEG - Master in Actuarial Science

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1. In a certain train station, it is known that male passengers arrive at a Poisson process $\{N_m(t)\}_{t\geq 0}$ with rate 3 per minute, while female passengers arrive at a Poisson processes $\{N_f(t)\}_{t\geq 0}$ with rate 2 per minute. The processes $\{N_m(t)\}_{t\geq 0}$ and $\{N_f(t)\}_{t\geq 0}$ are assumed to be independent.

   (a) Find the probability that, in a 3-minute period, the combined total number of passengers is at least 11. Give reasons for your answer.

   (b) Find the probability that, again in a 3-minute period, there are three female arrivals provided that there are 11 arrivals. Give reasons for your answer.

2. An insurance company receives claims according to a Poisson process with rate = 4 per day. Assume that the claim sizes are i.i.d. and take the values 200, 300, 400, and 500 with equal probabilities (= 1/4).

   (a) What is a distribution of a random variable which represents a number of claims of size 200 within two consecutive days ?

   (b) Consider again two consecutive days. What is the probability that during the first day there were 2 claims of size 500 given that there were 3 claims in total within two days ?

   (c) What is the probability that a third claim of size 500 arrives earlier than a third claim of a smaller size ?

   (d) What is the probability that, within one day, a total claim size of all claims is less than 450 ?

3. Calls arrive at a switchboard as a Poisson process with rate = 4 per minute. Find the probabilities that, with respect to any fixed origin in time, the twelfth subsequent call arrives:

   (a) before two minutes have elapsed

   (b) after three minutes and before five minutes have elapsed.

4. Consider a portfolio of drivers and suppose that each driver may be classified as bad driver or good driver. The number of accidents reported by a bad driver follows a Poisson process with intensity 0.5 per year, while a good driver reports accidents according to a Poisson process with intensity 0.1 per year. Suppose that 80% of the drivers in the portfolio are good, while 20% are bad. Let $N(t)$ the number of accidents reported by time t by a driver chosen randomly from the portfolio of drivers. Let $N(0) = 0$, $p_k(t) = \Pr\{N(t) = k\}$ and $p_{k,k+n}(s,t) = \Pr\{N(t) - N(s) = n|N(s) = k\}$.

   (a) How do you classify the process $\{N(t)\}_{t\geq 0}$?

   (b) Are the increments of the process stationary? Justify.

   (c) Are the increments of the process independent? Justify.

   (d) Is the process homogeneous? Justify.

   (e) Determine $p_k(t)$ and $p_{k,k+n}(s,t)$.

   (f) Determine $p_2(2)$.

   (g) Determine $p_{1,2}(1,3)$.

5. $\{N(t)\}_{t\geq 0}$, with $N(0) = 0$, is a birth process with $\lambda_k(t) = \frac{(1+k)\beta}{1+\beta t}$. Let $p_k(t) = \Pr\{N(t) = k\}$ and $p_{k,k+n}(s,t) = \Pr\{N(t) - N(s) = n|N(s) = k\}$.

   (a) Prove that $p'_{k,k}(s,t) = -\lambda_k(t)p_{k,k}(s,t)$, $k = 0, 1, ...$ and $p'_{k,k+n}(s,t) = -\lambda_{k+n}(t)p_{k,k+n}(s,t) + \lambda_{k+n-1}(t)p_{k,k+n-1}(s,t)$ (with initial conditions $p_{k,k}(s,s) = 1$ and $p_{k,k+n}(s,s) = 0$, for $n \geq 1$) (The derivatives are with respect to t).

   (b) Prove that $p'_0(t) = -\lambda_0(t)p_0(t)$ and that $p'_n(t) = -\lambda_n(t)p_n(t) + \lambda_{n-1}(t)p_{n-1}(t)$, $n = 1, 2, ...$ (with the initial conditions $p_0(0) = 1$ and $p_n(0) = 0$, for $n \geq 1$).

   (c) Show that $p_k(t) = \frac{1}{1+\beta t}\left(\frac{\beta t}{1+\beta t}\right)^k$, $k = 0, 1, ...$ is the solution to the system of equations deduced in b).

(d) Show that $p_{k,k+n}(s,t) = \binom{k+n}{n} \left(\frac{\beta(t-s)}{1+\beta t}\right)^n \left(\frac{1+\beta s}{1+\beta t}\right)^{k+1}$, $n = 0, 1, 2, ..., k = 0, 1, 2, ...$ is the solution to the system of equations deduced in a).

(e) Considering $\beta = 1$ determine $p_{1,3}(1, 2)$.

(f) Considering $\beta = 1$ determine $\Pr\{N(1) = 1, N(2) = 3\}$.